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On the Stability of Finite Plastic Deformations

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Summary

The stability of inelastic deformation and the formation of shear bands are examined by considering a recently developed gradient-dependent theory of elasto-viscoplasticity. The main issues addressed in the present work are the role of viscosity, inertia and higher order strain gradients in providing internal length scales, as well as the effect of back stress (deformation induced anisotropy) and plastic spin (texture development) on the structure of the shear bands in the post localization regime.

Introduction

The problems of constitutive modelling of finite plastic deformations and the associated phenomena of deformation pattern-formation and shear banding have been the subjects of interest among many investigators during the past two decades; recent reviews can be found in the works of Rice [1], Aifantis [2a,b] and Zbib and Aifantis [3a,b]. When examining these phenomena, the crucial questions that arise are the identification of the basic scaling mechanisms, the associated microstructural aspects (see for example, Korbel et al. [4]), the appropriate internal variables that a constitutive theory should include and the way the macroscopic constitutive response is related to the physical microprocesses (see for example, Aifantis [2] and Drucker [5]).

The stability of plastic flow and the shear banding phenomenon, in particular, have gained the attention of many researchers in both the metallurgical and mechanics communities [1-11]. While the main mathematical and physical aspects pertaining to the understanding of the mechanisms leading to the onset of the phenomenon have been examined, work on the characterization and evolution of shear bands including the determination of band widths and spacings has only recently begun. The main difficulty in this problem lies in the proper modelling of material behavior in the post-localization regime once the shear bands are formed. This can be attributed to the fact that in this regime the deformation field becomes highly inhomogeneous and, therefore, constitutive equations originally proposed for homogeneous or "near homogeneous" fields become insufficient. To overcome such difficulties, it was suggested in the work of Aifantis and co-workers [2,3, 12-18] to include the higher order strain gradients

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into the constitutive theory of plasticity to account for the heterogeneity of plastic flow and capture internal deformation wavelengths.

In the present work we first review the "scale-invariance" approach outlined in [2] and used in [3] to obtain physically based constitutive equations for the plastic stretching, back stress (modelling anisotropy) and plastic spin (modelling texture development). Such a theory may be viewed as a compromise between "multislip" crystal plasticity averaging models and purely continuum plasticity models. The theory is then modified to account for the heterogeneity of plastic flow occurring during the shear banding process. This, in fact, can be directly related to microstructural inhomogeneities resulting, for example, from tangled cell structures of dislocations, texture development, etc. This, in turn, leads to the development of strain heterogeneities causing the initiation of localized deformation bands. The macroscopic manifestation of such microscopic events is assumed here to be the development of higher order strain gradients which would enter into the form of macroscopic phenomenological theory. In fact, we retain the structure of the classical theory of plasticity, including the yield surface (for rate-independent materials), the normality condition and the flow rule, but introduce higher order strain-gradients into the equation for the flow stress.

In order to examine the role of higher order strain gradients on the material behavior, we consider a simplified one-dimensional problem and investigate the effect of inertia, viscosity and gradients on the structure of the shear band. It is shown that for the quasi-static loading case, higher order strain gradients are the only source to provide an internal length scale to the problem for both rate-independent and rate-dependent materials. For the dynamic loading case, however, it i shown that viscosity and inertia, together, are sufficient to provide a natural length scale to the problem without resorting to higher order strain gradients, but when viscosity is dropped, higher order gradients become again necessary.

In order to examine the effect of plastic spin and anisotropy on the development of shear bands we consider a two-dimensional problem. The problem is treated numerically using the finite element method. It is shown that for the case of shear banding from a ground-state of pure extension (no shearing), the plastic spin has no influence on the onset of instability, but it does affect the development of severe localization. In fact, an increase in the degree of plastic spin decreases "ductility," in the sense that the value of average strain to severe localization and "load collapse" decreases.

A Gradient-Dependent Theory of Elasto-Plasticity

We consider a continuous medium under dynamic loading conditions whose motion is governed by the linear momentum equation.

$$\mathrm{div}\, \boldsymbol{\sigma} = \rho\, \dot{\mathbf{v}}, \tag{1}$$

where σ is the Cauchy stress tensor, ρ is the mass density, and $\mathbf{v}$ is the velocity vector field. Assuming that the solid undergoes large deformations, we can decompose the deformation gradient $\mathbf{F}$ into elastic $\mathbf{F}^e$, plastic $\mathbf{F}^p$ and rotation $\mathbf{R}^m$ parts such that $\mathbf{F} = \mathbf{R}^m\, \mathbf{F}^e\, \mathbf{F}^p$ as proposed by Zbib and Aifantis [3a]. Following their method, one can then obtain the decomposition

$$\mathbf{D} = \mathbf{D}^e + \mathbf{D}^p\, , \quad \boldsymbol{\omega} = \mathbf{W} - \mathbf{W}^p, \tag{2}$$

where the stretching $\mathbf{D}$ is the symmetric part of the velocity gradient, $\mathbf{D}^e$ and $\mathbf{D}^p$ are the elastic and plastic stretching respectively, ω is the spin of the material (substructure), $\mathbf{W}$ is the antisymmetric part of the velocity gradient (spin of the continuum) and $\mathbf{W}^p$ is the plastic spin. The elastic strain rate $\mathbf{D}^e$ is given by Hooke's law, modified for large deformations, as

$$\mathbf{D}^e = [\mathbf{C}^e]^{-1} \overset{\circ}{\mathbf{S}}\ ; \ \overset{\circ}{\mathbf{S}} = \dot{\mathbf{S}} - \boldsymbol{\omega}\mathbf{S} + \mathbf{S}\boldsymbol{\omega}, \tag{3}$$

where $[\mathbf{C}^e]$ is the elasticity tensor and $\overset{\circ}{\mathbf{S}}$ is the corotational stress rate.

Using a scale-invariance argument and a maximization procedure, Aifantis and co-workers [e.g. 2,3] have been able to derive explicit expressions for the plastic stretching $\mathbf{D}^p$ and plastic spin $\mathbf{W}^p$ based on the process of crystal slip and dislocation glide. For kinematic hardening plasticity models it turns out that the plastic anisotropy can be represented by the back stress $\boldsymbol{\alpha}$ whose evolution is, in fact, determined by the plastic spin $\mathbf{W}^p$. As a special case of this formulation one obtains the following constitutive equations

$$\mathbf{D}^p = \frac{\dot{\bar{\gamma}}}{2\bar{\tau}}(\mathbf{S}' - \boldsymbol{\alpha}'),\ \overset{\circ}{\boldsymbol{\alpha}} = h\mathbf{D}^p - c\dot{\bar{\gamma}}\,\boldsymbol{\alpha};\ \overset{\circ}{\boldsymbol{\alpha}} = \dot{\boldsymbol{\alpha}} - \boldsymbol{\omega}\boldsymbol{\alpha} + \boldsymbol{\alpha}\boldsymbol{\omega};$$
$$\boldsymbol{\omega} = \mathbf{W} - \mathbf{W}^p,\ \mathbf{W}^p = \zeta(\boldsymbol{\alpha}\mathbf{D}^p - \mathbf{D}^p\boldsymbol{\alpha}), \tag{4}$$

where prime indicates deviatoric part while, h, c and ζ are material parameters. [As usual, $\dot{\bar{\gamma}} = (2\,\mathbf{D}^p \cdot \mathbf{D}^p)^{1/2}$ denotes the effective strain and $\bar{\tau} = (\mathbf{S}' \cdot \mathbf{S}'/2)^{1/2}$ the effective stress].

Equation $(4)_6$ suggests that the plastic spin is a measure of noncoaxiality between the back stress and the stretching. Its implication to large deformation has been examined in detail by the authors [3]. In particular, it was shown that $\mathbf{W}^p$ has a considerable effect on the development of axial stress (or strain) in a fixed (or free) end torsion of a cylindrical bar and that the value of the parameter ζ can be obtained from such torsion tests. [The results of Zbib and Aifantis [3] suggested that ζ has very small values; in fact, by defining $\zeta = a/\sigma_y$, where σ_y is the initial yield stress, they obtained, for example, $a = 4.5$ for AL-1100]. The implication of the plastic spin to localization is examined in the last section of this paper in connection with the numerical analysis.

Rate-Dependent Materials

The flow stress $\bar{\tau}$ appearing in equation (4), and its dependence on strain and strain rate determines whether the material is rate-independent or rate-dependent. In addition to this classical interpretation of the flow stress, it was proposed in [2,3] that higher order gradients of micro (dislocation densities) or macro (strain) variables be included in the respective constitutive equation in order to capture the details of deformation patterning and shear banding phenomena. For the case of macro shear bands, for example, the following gradient-dependent expression for the flow stress was proposed

$$\bar{\tau} = \bar{\tau}_H - \bar{\tau}_I\, ; \; \bar{\tau}_H = \kappa(\bar{\gamma}, \dot{\bar{\gamma}}),\; \bar{\tau}_I = C_1 \nabla^2 \bar{\gamma} + C_2\, |\nabla \bar{\gamma}|^2, \tag{5}$$

where $\kappa\,(\bar{\gamma}, \dot{\bar{\gamma}})$ is the usual homogeneous stress and $C_1\,(\bar{\gamma})$ and $C_2\,(\bar{\gamma})$ are the so-called gradient coefficients. This simple expression provides an internal length to the theory of plasticity and, hence, makes possible the investigation of pattern-forming instabilities in the deformation field.

Upon combining equations (3) and (4) with (2) we obtain the following gradient-dependent constitutive equation for rate-dependent elastic-viscoplastic materials with isotropic/kinematic hardening

$$\hat{\mathbf{S}} = [\mathbf{C}^e]\mathbf{D} - \frac{1}{2\mu}[\mathbf{C}^e](\mathbf{S}' - \boldsymbol{\alpha}'); \qquad \mu = \mu_H - \mu_I\, ; \; \mu_H = \frac{\bar{\tau}_H}{\dot{\bar{\gamma}}},\;\; \mu_I = \frac{\bar{\tau}_I}{\dot{\bar{\gamma}}}. \tag{6}$$

where μ can be defined as the nonlinear total viscosity, with a homogeneous part μ_H and an inhomogeneous part μ_I.

When elastic effects are neglected ($\mathbf{D}^e$=0) for large plastic deformations, then we obtain the following constitutive relation for a gradient-dependent viscoplastic material with isotropic/kinematic hardening

$$\mathbf{S} = 2\mu\mathbf{D} + \boldsymbol{\alpha} - p\mathbf{1}; \tag{7}$$

where $p = -\,\mathrm{tr}\mathbf{S}/3$ is the hydrostatic pressure. This equation is reminiscent to that describing a Newtonian viscous fluid (without α and a gradient-dependent viscosity).

Rate-Independent Materials

The dependence of the flow stress κ on $\dot{\bar{\gamma}}$ indicates viscoplastic behavior. Plastic behavior is obtained by dropping in (5) the dependence on $\dot{\bar{\gamma}}$. In this case $\dot{\bar{\gamma}}$ and the condition of loading or unloading are obtained from the yield function F and the consistency condition $\dot{F} = 0$. The loading-unloading conditions are given by

$$F=\sqrt{J_2'}-\bar{\tau}, \quad \begin{cases} F=0 \text{ and } \dot{\bar{\gamma}}>0 \text{ or } \dot{\bar{\gamma}}=0 - \\ \qquad \text{loading or neutral loading} \\ F=0 \text{ and } \dot{\bar{\gamma}}<0 - \text{ unloading} \end{cases} \tag{8}$$

where $J_2' = (\mathbf{S}'-\boldsymbol{\alpha}')\cdot(\mathbf{S}'-\boldsymbol{\alpha}')/2$. The consistency condition $\dot{F}=0$ then yields

$$C_1\nabla^2\dot{\bar{\gamma}}+2C_2\nabla\bar{\gamma}\cdot\nabla\dot{\bar{\gamma}}-\bar{h}\dot{\bar{\gamma}}=-\frac{(\mathbf{S}'-\boldsymbol{\alpha}')\cdot\dot{\mathbf{S}}'}{2\bar{\tau}}; \tag{9}$$

where the "effective" hardening modulus $\bar{h}$ is given by

$$\bar{h}=\frac{d\bar{\tau}_H}{d\bar{\gamma}}+\frac{h}{2}-\frac{c}{2\bar{\tau}}\boldsymbol{\alpha}'\cdot(\mathbf{S}'-\boldsymbol{\alpha}')-\frac{dC_1}{d\bar{\gamma}}\nabla^2\bar{\gamma}-\frac{dC_2}{d\bar{\gamma}}\nabla^2\bar{\gamma}-\frac{dC_2}{d\bar{\gamma}}|\nabla\bar{\gamma}|^2. \tag{10}$$

Relation (9) is a differential equation for the loading index $\dot{\bar{\gamma}}$. This is a major difference between the present theory and the classical theory of plasticity. The loaning index now should be treated as an independent variable governed by equation (9). [This could be treated numerically and incorporated in usual finite element programs where equations (9) can be solved independently at each integration increment].

<u>The One-Dimensional Problem: Length and Time Scales</u>

In order to illustrate the implications of the theory outlined in the previous section to the shear banding problem, we consider the simple shearing of an infinite block in the x direction. We assume that the state of stress is pure shear such that $S_{xy}=S_{yx}=\tau(t)$ are the only nonzero components of $\mathbf{S}$. Similarly, $\dot{\epsilon}_{xy}=\dot{\epsilon}_{yx}=\dot{\gamma}/2$ are the only nonzero components of $\mathbf{D}$. Moreover, we assume isotropic hardening with no back stress. Then it can be shown that equations (1)-(3) reduce to

$$\frac{\partial\tau}{\partial y}=\rho\dot{v}\ , \quad \dot{\tau}=G(\dot{\gamma}-\dot{\gamma}^p), \tag{11}$$

where G is the elastic shear modulus, $\dot{\gamma}^p$ is the plastic strain rate, and v is the velocity field in the x direction. Various classes of material behavior are considered below.

Elastic-Plastic Materials

a) No gradients: $C_1=C_2=0$.

For this case we drop the dependence on $\dot{\bar{\gamma}}$ from equation (5) and obtain $\dot{\gamma}^p=\dot{\tau}/H$, where $H=\partial\kappa/\partial\gamma^p$. Then equations (11) (with $\dot{\gamma}=v_y$) yield a rather familiar partial differential equation

$$\ddot{v}=c^2v_{yy}+\frac{1}{\rho}\left(\frac{GH}{H+G}\right)_y v_y\ ; \quad c^2=\frac{GH}{\rho(H+G)}, \tag{12}$$

where the subscript y indicates partial derivative. The type of this differential equation is governed by the sign of H. If H > 0 the equation is hyperbolic and travelling waves with a speed equal to c exist. However, when H < 0 the equation becomes elliptic and wave trapping occurs [19]. In this respect the

problem is "ill-posed" due to the change of type occurring in the governing differential equation as the material enter the softening regime (H < 0).

b) With gradients: C_1 = constant, C_2 = constant.

The problem of "ill-posedness" is eliminated by introducing higher order gradients. This can be seen by considering equation (5) (with no dependence on $\dot{\gamma}$) which upon neglecting elasticity from $(11)_2$ $(\dot{\gamma} = \dot{\gamma}^p)$ and combining with $(11)_1$ gives

$$\rho\ddot{u} = Hu_{rr} - C_1 u_{rrrr} - 2C_2 u_{rr} u_{rrr} \tag{13}$$

with u denoting the displacement. Now the character of this differential equation is independent of H and is determined by C_1 which is a constant. Therefore, the system does not change type. Moreover, equation (13) can be nondimensionalized by introducing the following "natural" time (η) and length (l) scales

$$\eta = \sqrt{\frac{\rho C_1}{H^2}}, \quad l = \sqrt{\frac{C_1}{|H|}}. \tag{14}$$

Then with the nondimensional variables

$$\bar{y} = \frac{y}{l}, \quad \bar{u} = \frac{u}{l}, \quad \bar{t} = \frac{t}{\eta}, \tag{15}$$

equation (13) becomes

$$\bar{u}_{,\bar{t}\bar{t}} = \bar{u}_{\bar{y}\bar{y}} - \bar{u}_{\bar{y}\bar{y}\bar{y}\bar{y}} - \left(\frac{2C_2}{C_1}\right)\bar{u}_{\bar{y}\bar{y}}\bar{u}_{\bar{y}\bar{y}\bar{y}} \tag{16}$$

This suggests that the width of the shear band (w) is proportional to l, i.e.,

$$w = \left(\frac{C_1}{|H|}\right)^{1/2}. \tag{17}$$

When H > 0, it is shown in [3] that the homogeneous state is stable and, therefore, shear bands do not initiate. They do, however, when $H \le 0$; i.e. according to (17) at H = 0, $w \to \infty$, and as $|H| \to \infty$ (in the softening regime) $w \to 0$, corresponding to severe localization. This is consistent with the quantitative result given in [12,13] where equation (16) is solved numerically with the inertia term on the left hand side set equal to zero. The result for 70/30 brass is shown in Figure 1 and compared with the experimental ones obtained recently by Joshi et al. [20]. Note that such nondimensionalization cannot be performed on equation (12) for which a "natural" length scale cannot be defined.

Elastic-viscoplastic Materials

For this case we drop the gradient terms from equation (5) and assume that $(5)_2$ can be inverted to formally read

$$\dot{\gamma}^p = f(\tau, \gamma^p). \tag{18}$$

Then equation (18) can be substituted into $(11)_2$ which combined with $(11)_1$ yields

$$\ddot{v} + G\frac{\partial f}{\partial \tau}\dot{v} = c^2 v_{yy} - c^2 \frac{\partial f}{\partial \gamma^p}\gamma^p_y , \quad c^2 = \frac{G}{\rho}. \tag{19}$$

This partial differential equation is always hyperbolic since the speed of the propagating wave c is real and equal to the speed of the elastic wave (regardless of the sign of the hardening modulus $\partial f/\partial \tau$). This result for elastic-viscoplastic materials is different than that for the elastic-plastic materials given by (12) where the wave speed and the character of the governing differential equation is determined by the sign of H. Moreover, in this case one can also define "natural" length and time scales as

$$\eta = \left(G\frac{\partial f}{\partial \tau}\right)^{-1} , \quad l = c\eta , \tag{20}$$

which with the aid of the nondimensional variables (15) can reduce equation (19) into the form

$$\bar{v}_{\bar{t}\bar{t}} + \bar{v}_{\bar{t}} = \bar{v}_{\bar{y}\bar{y}} - \left(\eta\frac{\partial f}{\partial \gamma^p}\right)\gamma^p_{\bar{y}} , \tag{21}$$

where $\bar{v} = v/c$. This nondimensionalization implies that the band width is proportional to l or in view of (20) and $(19)_2$,

$$w = \frac{1}{(\partial f/\partial \tau)\sqrt{G\rho}}. \tag{22}$$

In the static case, however, equation $(11)_1$ implies that τ is homogeneous in y and, therefore, one has only equation (18) which has no length scale. Hence, higher order gradients should be considered and the original equation (5) yields

$$\tau(t) = \kappa(\gamma,\dot{\gamma}) - C_1\gamma_{yy} - C_2\gamma_y^2 \tag{23}$$

where $\tau(t)$ is a function of time only since $\partial\tau/\partial y = 0$ for the static case. Equation (23) has been solved in [13] numerically where it is shown that the band width is independent of the mesh size and depends only on the values of C_1 and C_2 as can be seen from Figure 2.

<u>Effect of Plastic Spin: Numerical Analysis</u>

For the numerical analysis considered here, we neglect elasticity and assume that the material is viscoplastic exhibiting kinematic/isotropic strain hardening and softening according to the constitutive equations (4), (5) and (7). This, in turn, results into a convenient viscous flow FEM formulation [21] with a nonlinear secant stiffness matrix, and eliminates the severe numerical limitations associated with usual elasto-plastic formulations. In fact, equation (7) and the momentum equation (1), with the inertia term neglected, along with the usual finite element formulation yield [21]

$$[\mathbf{K}(\mathbf{X},\mathbf{V},\bar{\tau}_{,i})]\mathbf{V} = \mathbf{F}_s + \mathbf{F}_\alpha, \tag{24}$$

where **K** is the secant stiffness matrix which is a nonlinear function of the nodal velocity vector **V**, nodal position vector **X** and strain gradients $\bar{\tau}_{,i}$. $\mathbf{F}_s$ is the boundary force vector and $\mathbf{F}_\alpha$ is the body force vector arising from the back stress α. The nonlinear dependence on **V** arises from the fact that the viscosity μ is nonlinearly related to $\dot{\bar{\gamma}}$.

Equation (24) is solved for V using the secant method. An explicit integration scheme with a constant time step using the Newton's forward method is used to integrate the velocity and the evolution equation for the back stress $(4)_2$. This integration algorithm along with the secant method to handle nonlinearities in V and by continuously updating the geometry at each time increment to handle the geometric nonlinearities, proved to be very efficient and produced stable results.

For the numerical analysis we use the following power law expression to model the isotropic hardening and softening

$$\kappa(\bar{\gamma},\dot{\bar{\gamma}}) = \tau_0\left(\frac{\dot{\bar{\gamma}}}{\dot{\gamma}_0}\right)^m\left(1+\frac{\bar{\gamma}}{\gamma_0}\right)^n\left(1+\frac{\bar{\gamma}}{\gamma_1}\right)^\nu , \tag{25}$$

where τ_0 is the initial yield stress, $m > 0$ and $n > 0$ are the strain rate sensitivity and strain hardening exponents respectively, $\nu < 0$ is the "strain softening" exponent, and $\dot{\gamma}_0, \gamma_0$ and γ_1 are material constants [21]. For a typical structural steel we use the values $\tau_0 = 400/\sqrt{3}$ MPa, $h = 40^{\circ}$ MPA, $C = 4.0$, $n = 0.1$, $m = 0.01$, $\nu = -2.0$, $\gamma_0 = 2\times10^{-3}$, $\dot{\gamma}_0 = 1.0\ \mathrm{sec}^{-1}$, $\gamma_1 = 2.0$. It is noted that for the present case the gradient effects are dropped by setting $C_1 = C_2 = 0$.

The numerical method is used to analyze the development of shear bands in a plane-strain tensile test. Due to symmetry, only one-quarter of the tensile specimen is examined. The one-quarter is meshed into 350 rectangular elements each consisting of four cross-triangular elements. The specimen is deformed in tension with a constant cross-head speed of V = 0.01 cm/sec. A length to width aspect ratio of 3 is used. A brief summary of the results are shown in Figures 3-7 which are sufficient to illustrate the effects of viscosity and plastic spin on the shear bands development. Figure 3 shows the mesh of the specimen with two "cross" shear band clearly formed. A closer look at the bands is given in Figure 4 where only one-quarter of the specimen is shown. It can be seen from the figure that the width of the band spans over at least two elements. The corresponding strain contours are shown in Figure 5 indicating the severity of the strain in the band and the fact that the strain field varies only across the band with constant contours along it. The effect of the plastic spin is examined by varying the value of $\zeta = a/\sigma_0$. [When $\zeta = 0$ the corotational rate given by $(4)_3$ reduces to the usual Jaumman rate]. Figure 6 shows P/P_Y versus U/L for various values of a, where P is load, P_Y is yield load, U is elongation and L is specimen length. The sharp drop in the load-displacement curve for U/L > 0.2 corresponds to the development of the shear band. The band seems to initiate around U/L ~ 0.2. It seems that the parameter a has no influence on the curve until the band becomes severe around U/L ~ 0.25. This is expected since the plastic spin influences the shearing mode which (for the present problem of initially

stretching model) becomes significant only when the band forms. The result, thus, suggests that the plastic spin does not influence the initiation of shear bands from a stretching mode (see also Tvergaard and Giessen [22]), it may however from a shearing mode. An increase in the degree of the plastic spin seems to decrease "ductility" in the sense that the value of the average strain to final load "collapse" decreases as a increases.

Finally, Figure 7 shows the variation of the kinetic energy E/E_o, where $E_o = mV^2/2$, m is the total mass, and V is the imposed boundary velocity. It can be seen from the figure that the kinetic energy remains almost constant in the early stages of deformation then increases steadily until the shear band initiates around $U/L \cong 0.2$, resulting to a sharp increase in the kinetic energy. Thereafter, the kinetic energy reaches another "steady" state once the band is completely formed. Note that E/E_o is slightly less than 1.0. This is expected since the material outside the band moves rigidly with velocity V.

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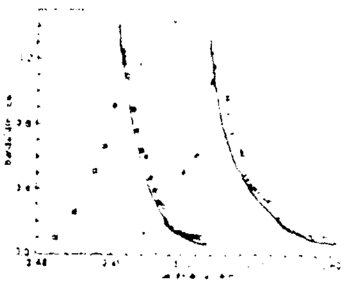

Figure 1. Shear bandwidth versus Strain in 70/30 Brass,
A) rolling angle of 45°,
B) rolling angle up 0°.

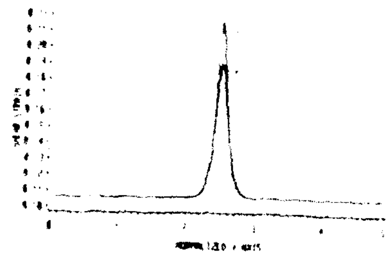

Figure 2. Effect of mesh size
A) $\Delta\gamma$ = 0.1, B) $\Delta\gamma$ = 0.5.

Figure 3. Deformed Mesh.

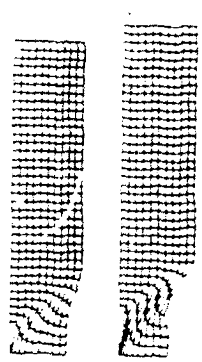

Figure 4. Deformed meshes.

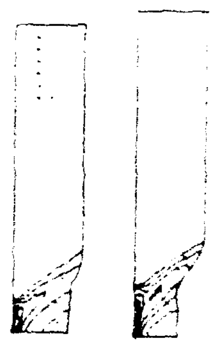

Figure 5. Strain Contours.

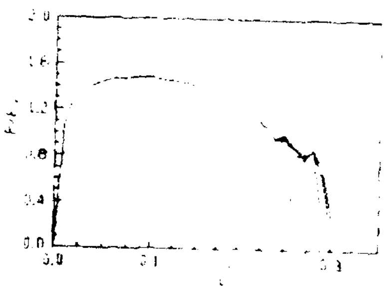

Figure 6. Effect of plastic spin on Load-extension curve.

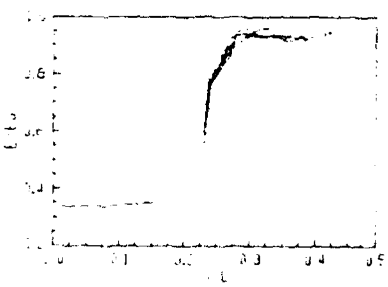

Figure 7. Effect of plastic spin on Kinetic energy.